UNIVERSITY OF CALICUT

P.G. ENTRANCE EXAMINATION

Subject : STATISTICS

MODEL QUESTION PAPER

Time: 2 Hours Max. Marks: 100

Instruction

Answer **all** *questions.*

PART - A

Each question carries **one** *mark.*

1) The function $f(x) = |x-1|, \ -\infty < x < \infty$ is:

(A) differentiable every where (B) continuous every where

(C) differentiable but not continuous (D) none of these

2) If $y = e^{-\frac{x^2}{2}}$, then $\frac{dy}{dx}$ is:

(A) $xe^{-\frac{x^2}{2}}$ (B) $-2xe^{-\frac{x^2}{2}}$ (C) $-xe^{-\frac{x^2}{2}}$ (D) $2xe^{-\frac{x^2}{2}}$

3) The value of $\Gamma(2.5)$ is:

(A) 1.87 (B) 0.75 (C) $0.25\sqrt{\pi}$ (D) $0.75\sqrt{\pi}$

4) Which of the following is a monotonically increasing function:

(A) $f(x) = x^2, \ -1 \leq x \leq 1$ (B) $f(x) = -x, \ 0 \leq x \leq 1$

(C) $f(x) = \frac{1}{x}, \ 1 \leq x \leq 2$ (D) $f(x) = (x-1)^2, \ 1 \leq x \leq 2$

5) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if:

(A) $p > 1$ (B) $p \geq 0$ (C) $p = 1$ (D) none of these.

6) Which of the following set is a countable subset of $\mathcal{R}$, the set of real numbers:

(A) the set of irrational numbers in $\mathcal{R}$ (B) the set of rational numbers in $\mathcal{R}$

(C) both (A) and (B) (D) none of these sets are countable

7) The value of $\lim_{x \to 1} \frac{x-1}{|x-1|}$ is:

(A) 1 (B) -1 (C) 0 (D) doesn't exist

8) If $A = \begin{bmatrix} \alpha & 1 \\ 1 & \alpha \end{bmatrix}$, and $|A^2| = 64$, then α is:

(A) ± 1 (B) ± 2 (C) ± 3 (D) ± 4

9) If $\mathbf{x}=(x_1, x_2, \ldots, x_n)$ is a set of n observations, then the Harmonic Mean of $\mathbf{x}$ is:

(A) the reciprocal of the AM of the given observation

(B) the reciprocal of the AM of the reciprocals of the observations

(C) the AM of the reciprocals of the observations (D) none of these

10) Which measure of central tendency is least effected by extreme observations?

(A) AM (B) GM (C) Median (D) HM

11) Which measure of central tendency can help the manufacturers of ready made garments in arriving at the desired size of a new ready made garments:

(A) AM (B) GM (C) Median (D) Mode

12) The correct relationship between AM, GM and HM is:

(A) AM $\geq$ GM $\geq$ HM (B) GM $\geq$ HM $\geq$ AM

(C) HM $\geq$ GM $\geq$ AM (D) AM $\geq$ HM $\geq$ GM

13) Sum of the squares of deviations of a set of observations is minimum, when the deviations are taken from their:

(A) mean (B) median (C) mode (D) zero

14) If $'a'$ is the actual value and $'e'$ is the estimated value, then the relative error is:

(A) $\frac{a}{e}$ (B) $\frac{a-e}{e}$ (C) $|\frac{a-e}{e}|$ (D) $|\frac{a-e}{a}|$

15) The component of a time series attach to long term variations is termed as:

(A) cyclic variation (B) secular trend (C) irregular variation (D) all the above

16) To which component of time series, the term recession is attached:

(A) secular trend (B) seasonal varication (C) cyclic variation (D) random variation

17) Index number is a:

(A) measure of relative changes (B) special type of an average

(C) percentage of relative changes (D) all the above

18) For a population Net Reproduction Rate (NRR) is equal to one, then we conclude that:

(A) female population exactly replace itself (B) population remains constant

(C) there is no mortality in female birth and their child bearing age (D) all the above

19) If b_{yx} and b_{xy} are two regression coefficients, then they have:

(A) same sign (B) opposite sign (C) either same or opposite signs (D) nothing can be say

20) The correlation coefficient between X and Y is 1, then the angle between two regression lines is:

(A) 30 (B) 90 (C) 0 (D) 60

21) The geometric mean of two regression coefficients b_{yx} and b_{xy} is:

(A) r (B) r^2 (C) 1 (D) none of these

22) If $B \subset A$, the conditional probability $P(A|B)$ is:

(A) 0 (B) 1 (C) $\frac{P(B)}{P(A)}$ (D) $\frac{P(A)}{P(B)}$

23) Given $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$, and $P(A|B) = \frac{1}{8}$. Then $P(B|A)$ is :

(A) $\frac{1}{8}$ (B) $\frac{1}{16}$ (C) $\frac{1}{32}$ (D) none of these

24) If A, B, C are any three events, which of the following is always true:

(A) $P(AB) = P(A)P(A|B)$ (B) $P(ABC) = P(A)P(B|A)P(AB|C)$

(C) $P(ABC) = P(AB)P(AB|C)$ (D) $P(ABC) = P(A)P(B|A)P(C|AB)$

25) If the joint pdf of two random variables X and Y are given by

$$f(x,y) = \begin{cases} x+y, & 0 \le x, y \le 1 \\ 0, & \text{otherwise,} \end{cases}$$

the marginal distribution of X is:

(A) $f_X(x) = x + \frac{1}{4}$, $0 \le x \le 1$ (B) $f_X(x) = x + \frac{1}{2}$, $0 \le x \le 1$

(C) $f_X(x) = x + y + 1$, $0 \le x, y \le 1$ (D) none of these

26) The joint probability mass function of two random variables X and Y is given below.

	-1	+1
0	$\frac{1}{8}$	$\frac{3}{8}$
1	$\frac{2}{8}$	$\frac{2}{8}$

The correlation coefficient between X and Y is:

(A) 0 (B) $-\frac{1}{8}$ (C) $-\frac{1}{\sqrt{15}}$ (D) $-\frac{1}{4}$

27) What is the value of k for which the function $f(x)$ given below is a probability density function:

$$f(x) = \begin{cases} 3kx^2, & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

(A) 1 (B) 3 (C) 9 (D) $\frac{1}{3}$

28) If X and Y are two random variables, then which of the following is always true:

(A) $E(XY)^2 = E(X^2)E(Y^2)$ (B) $E(XY)^2 \leq E(X^2)E(Y^2)$

(C) $E(XY)^2 \geq E(X^2)E(Y^2)$ (D) none of these

29) If $f(x, y)$ is the joint distribution of X and Y, then $E(Y|X = x)$ is known as:

(A) regression curve of X on Y (B) regression curve of Y on X

(C) both (A) and (B) (D) none of these

30) If X is a gamma variate with parameter λ, then the moment generating function of X is :

(A) $(1-t)^{-\lambda}$, $|t| < 1$ (B) $(1+t)^{-\lambda}$, $|t| < 1$ (C) $(1-t)^{\lambda}$, $|t| > 1$ (D) none of these

31) If X is normally distributed with mean zero and unit variance, then the variance of X^2 is:

(A) 1 (B) 0 (C) 4 (D) 2

32) The probability density curve of chi-square distribution in respect of symmetry is:

(A) symmetrical (B) negatively skewed

(C) positively skewed (D) none of these are correct

33) If X_1 and X_2 are two independent standard normal random variables, then the distribution of $\frac{\sqrt{2}\ X_2}{\sqrt{X_1^2+X_2^2}}$ is:

(A) chi-square distribution (B) student's-t distribution

(C) F-distribution (D) none of these

34) If an estimator T_n of a population parameter θ converges in probability to θ as $n \to \infty$, then T_n is said to be:

(A) sufficient estimator (B) consistent estimator

(C) efficient estimator (D) unbiased estimator

35) Let (2,0,5) be an observed sample from an exponential population with probability density function $f(x) = \frac{1}{\theta}e^{-\frac{x}{\theta}}$, $x \geq 0$, $\theta > 0$. Then the moment estimator based on the given sample is:

(A) $\frac{7}{3}$ (B) $\frac{7}{2}$ (C) $\frac{3}{7}$ (D) $\frac{4}{3}$

36) Let $(X_1, X_2, \ldots, X_n)$ be a random sample from geometric distribution with probability mass function $P(X = x) = p(1-p)^{x-1}$; $x = 1, 2, 3, \ldots;\ 0 < p < 1$. Then the maximum likelihood estimate of p is:

(A) $\overline{X}$ (B) $\Sigma_{i=1}^{n} X_i$ (C) $\frac{1}{\overline{X}}$ (D) $\frac{1}{\Sigma_{i=1}^{n} X_i}$

37) For a statistical test procedure, the level of significance is the probability of:

(A) Type I error (B) Type II error

(C) both Type I and Type II error (D) none of these

38) Which of the following result provides sufficient statistic through factorization method:

(A) Crammer-Rao inequality (B) Rao-Blackwell theorem

(C) Fisher-Neymann theorem (D) Baye's theorem

39) Which of the following is always true:

(A) MLE is unbiased (B) MLE is consistent (C) MLE is unique (D) none of these

40) Which of the following is not a contrast for treatments T_1, T_2, T_3?

(A) $T_1 + 2T_2 - T_3$ (B) $2T_1 - T_2 - T_3$ (C) $T_1 + T_2 - 2T_3$ (D) $3T_1 - 2T_2 - T_3$

41) Which of the following design is not based on all the three basic principles of experimentation?

(A) LSD (B) RBD (C) CRD (D) none of these

42) The regression between the central mortality rate "m_x"and force of mortality "μ_x" is:

(A) $m_x = \mu_x + \frac{1}{2}$ (B) $m_x = \frac{1}{2}\mu_x + 1$ (C) $m_x = \frac{1}{2}\mu_x + \mu_{x+1}$ (D) none of these

43) The variation in the items produced in a factory may be due to:

(A) chance causes (B) assignable causes

(C) both chance and assignable causes (D) none of theses

44) Control limits for R chart with usual notation are:

(A) UCL=D_4R, C.L= R and LCL=D_3R (B) UCL=$D_4\overline{R}$, C.L=$\overline{R}$ and LCL= $D_3\overline{R}$

(C) UCL=$D_4\overline{R}$, C.L=$\overline{R}$ and LCL= $D_4\overline{R}$ (D) none of these

45) If μ and σ are the mean and standard deviation, then the control limits $\mu - 3\sigma$ and $\mu + 3\sigma$ are known as:

(A) natural control limits (B) modified control limits

(C) specified control limits (D) none of these

46) Probability of drawing a unit to the sample at each selection remains same in:

(A) SRSWR (B) SRSWOR (C) both (A) and (B) (D) none of (A) and (B)

47) Which year was celebrated as the international year of Statistics by the United Nations:

(A) 2012 (B) 2013 (C) 2014 (D) 2015

48) If x_j is a basic variable in a simplex table, then the relative cost $z_j - c_j$ is:

(A) positive (B) negative (C) zero (D) none of these

49) Which of the following method is used for solving assignment problem?

(A) reduced matrix method (B) north-west corner method

(C) Hungarian method (D) none of these

50) The distinguishing feature of a linear programming problem is:

(A) it has single objective function and constrain

(B) value of the decision variable is non-negative

(C) relationship among all variables is linear

(D) all the above

PART - B

Each question carries **two** *marks.*

51) If $f(x) = x + 5,\ 0 \leq x \leq 10,$ and $\mathcal{P} = \{0, 2, 5, 6, 8, 10\}$ is a partition of $[0, 10]$, then $U(\mathcal{P}, f) - L(\mathcal{P}, f)$ is:

(A) 22 (B) 8 (C) 100 (D) 111

52) If the function given below is continuous at 2, then the value of a is:

$$f(x) = \begin{cases} \frac{ax^3+x^2-(a+2)x+a}{x-2} & for \quad x \neq 2 \\ \\ 2 & for \quad x = 2 \end{cases}$$

(A) 1 (B) 8 (C) 0 (D) none of these

53) If $f(x)$ is differentiable for all x, then the values of a and b respectively are:

$$f(x) = \begin{cases} ax & for \quad x < 2 \\ \\ ax^2 - bx + 3 & for \quad x \geq 2 \end{cases}$$

(A) 1,1 (B) $\frac{3}{4}$, 2 (C) $\frac{3}{4}$, $\frac{4}{9}$ (D) $\frac{3}{4}$, $\frac{9}{4}$

54) The mean and variance of the first **10** natural numbers are:

(A) 5.5, 38.5 (B) 5.5, 8.25 (C) 55, 8.25 (D) none of these

55) A man goes from his house to his office at the speed of 20 km/hr and returns from his office to home at a speed of 30 km/hr. His average speed in km/hr is:

(A) 24 (B) 25 (C) 30 (D) $10\sqrt{6}$

56) If $\rho_{X,Y} = 0.5$, $Z_1 = 2X - 4$ and $Z_2 = 3 - 2Y$. Then the correlation between Z_1 and Z_2 is:

(A) 1 (B) 0.5 (C) -0.5 (D) none of these

57) The two regression equations in an analysis are obtained as: $2x+3y-26 = 0$ and $x+6y-31 = 0$. The correlation coefficient r_{xy} is:

(A) 0.25 (B) -0.25 (C) 0.50 (D) -0.50

58) Let A, B and C are three mutually independent events with $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(C) = \frac{1}{4}$. Then $P(A \cup B \cup C)$ is:

(A) $\frac{3}{4}$ (B) $\frac{2}{3}$ (C) $\frac{27}{24}$ (D) none of these

59) Which of the following is not a probability density function?

(A) $f(x) = \begin{cases} \frac{1}{2}, & -1 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$ (B) $f(x) = \begin{cases} 2x, & -1 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$

(C) $f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$ (D) none of the above is a pdf

60) In answering a question in a multiple choice test, a student either knows the answer or guesses the answer with equal probability. While guessing, the student is equally likely to select any one of the four choices of answers in the question. Then the conditional probability that the student really knows the answer given that he/she answered the question correctly is:

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{4}{5}$ (D) 1

61) Two dice are thrown simultaneously, let X be a random variable representing the sum of the topmost faces on these dice. What is E(X)?

(A) 6 (B) $\frac{21}{36}$ (C) 7 (D) none of these

62) If X is a $U(0,1)$ random variable and $Y = -2 \log X$, then the distribution of Y is:

(A) $F -$ distribution (B) $t -$ distribution

(C) exponential distribution (D) chi-square distribution

63) Let X_1, X_2, $X_3 \sim B(1,\theta)$, define $Y_i = 1 - X_i$, $i = 1, 2, 3$. Then the distribution of $Y_1 + Y_2 + Y_3$ is:

(A) $B(3, 1-\theta)$ (B) $B(1, 1-\theta)$ (C) $B(3,\theta)$ (D) $B(1,\theta)$

64) The random variable X has a probability density function

$$f(x) = \frac{1}{\sqrt{32\pi}} e^{-\frac{(x+7)^2}{32}}, \; -\infty < x < \infty.$$

Then the moment generating function of X is:

(A) e^{-7t+t^2} (B) e^{-t+8t^2} (C) e^{7t+6t^2} (D) e^{-7t+8t^2}

65) If X_1, X_2 are random variables following $N(0,1)$ distribution, which of the following has a chi-square distribution with one degree of freedom?

(A) $X_1^2 + X_2^2$ (B) $\frac{(X_1-X_2)^2}{2}$ (C) $\left(\frac{X_1-X_2}{2}\right)^2$

(D) none of the above has a chi-square distribution with one degree of freedom.

66) Let (X, Y) has the density

$$f(x,y) = \begin{cases} \frac{1}{\sqrt{2\pi x^2}} e^{-\frac{1}{2}(y-2-x)^2}, & \text{for} \quad 1 \le x < \infty,\ -\infty < y < \infty \\ 0, & \text{otherwise.} \end{cases}$$

Then $E(Y|X)$ is:

(A) $1+X$ (B) $2+X$ (C) $3-X$ (D) none of these

67) If $X_1, X_2, \ldots, X_n$ is a random sample of size n from $B(1,\theta)$ and $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, which of the following is not a correct statement:

(A) $\overline{X}_n$ is an estimator of θ which is unbiased and consistent

(B) $\overline{X}_n$ is a sufficient estimator of θ

(C) $\frac{n+1}{n}\ \overline{X}_n$ is a consistent estimator of θ (D) None of the above statement is correct

68) The mean square error of an estimator T_n of a parameter θ, can be expressed as:

(A) Bias+ Variance of T_n (B) (Bias+Variance of T_n)^2

(C) (Bias)^2 + Variance of T_n (D) Bias + (Variance of T_n)^2

69) Let $(X_1, X_2, \ldots, X_n)$ be a random sample of size n from $N(\mu, \sigma^2)$, where μ is known, then the maximum likelihood estimator of σ^2 is:

(A) $\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$ (B) $\frac{1}{n}\sum_{i=1}^{n}(X_i - \mu)^2$ (C) both (A) and (B) (D) none of these

70) For testing $H_0 : \lambda = 1$, against $H_1 : \lambda = 1.5$ based on a single observation X, from a population with pdf:

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0,\ \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$$

the critical region be '$X > 2$'. Then the size of the test is:

(A) 0 (B) $\frac{1}{e}$ (C) $e^{-1.5}$ (D) $\frac{1}{e^2}$

71) In SRSWOR of size n from a population of size N with usual notations the variance of an unbiased estimator of populations total is:

(A) $\frac{N-n}{Nn}S^2$ (B) $\frac{N(N-n)}{n}S^2$ (C) $\frac{n(N-n)}{N}S^2$ (D) none of these

72) A study of sampling methods was conducted in a population having 500 units. By total count it was obtained that $\overline{Y} = 49$ and $S^2 = 44.6$. In a simple random sampling with replacement,

how many sampling units approximately should be chosen for estimating $\overline{Y}$ with a permissible marginal error of 10% and 95% of confidence coefficient?

(A) 6 (B) 8 (C) 10 (D) 15

73) In an experiment to investigate the effect of relative humidity on abrusion resistance of leather cut from a rectangular pattern a 6×6 Latin Square was used to control possible variability due to row and column positions in the pattern. Six levels of relative humidity were 25%, 37%, 50%, 62%, 75% and 87%. What is the error degrees of freedom in this experiment?

(A) 5 (B) 6 (C) 20 (D) 35

74) In a Linear Programming Problem, if two constraints do not intersect in the positive quadrant of the graph, then:

(A) the solution is unbounded (B) the solution is infeasible

(C) one of the constraints is redundant (D) none of these

75) The optimal solution to the linear programming problem,

Maximize $z = 5x_1 + 3x_2$

subject to the constraints $2x_1 + x_2 \leq 1000, \;\; 0 \leq x_1 \leq 400, \;\; 0 \leq x_2 \leq 700$ is:

(A) $x_1 = 0, \; x_2 = 0$ (B) $x_1 = 400, \; x_2 = 0$

(C) $x_1 = 400, \; x_2 = 200$ (D) $x_1 = 150, \; x_2 = 700$

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