DEPARTMENT OF STATISTICS

UNIVERSITY OF CALICUT

MODEL QUESTION PAPER OF M.Phil ENTRANCE EXAMINATION

Time: 2 Hours Max.Marks: 100

Instructions:

Answer **all** *questions from* **PART A** *and* **any five** *questions from* **PART B**.

PART - A

Each question carries **two** *marks.*

1) If X and Y are i.i.d. r.v.s with $X-Y$ and $X+Y$ are independent. Then which of the following is always true?

(A) $X-Y$ has Cauchy (0,1) distribution (B) $X+Y$ has Cauchy (0,1) distribution

(C) $\frac{X}{X+Y}$ has $N(0,1)$ distribution (D) $X+Y$ has Normal distribution

2) Which of the following is not in one parameter exponential family?

(A) Binomial (n,p), n known, $0<p<1$ (B) Poisson (λ), $\lambda>0$

(C) Uniform $(0,\theta]$, $\theta>0$ (D) Gamma (α,β), α known, $\beta>0$

3) For a positive r.v X with finite first moment, which of the following is not true?

(A) $\text{E}(\sqrt{X}) \geq \sqrt{E(X)}$ (B) $\text{E}(\sqrt{X}) \leq \sqrt{E(X)}$

(C) $\text{E}(\frac{1}{X}) \geq \frac{1}{E(X)}$ (D) $\sqrt{E(X^2)} \leq \sqrt[3]{E(X^3)}$

4) If $\text{E}(X^2)<\infty$, then Var(X) is

(A) Var(E($X|Y$)) (B) E(Var($X|Y$)) (C) Var(E($X|Y$))+E(Var($X|Y$))

(D) Var(E($Y|X$))+E(Var($Y|X$))

5) If $X_{(1)},\ X_{(2)},\ X_{(3)}$ be the order statistics of a sample of size 3 from $U[0,1]$, then Var$(X_{(2)})$ is

(A) $\frac{1}{20}$ (B) $\frac{1}{10}$ (C) $\frac{2}{5}$ (D) $\frac{3}{10}$

6) Let X be a r.v. with pmf under H_0 and H_1 given by

x	1	2	3	4	5	6
$f_0(x)$	0.01	0.01	0.01	0.01	0.01	0.95
$f_1(x)$	0.05	0.04	0.03	0.02	0.01	0.85

If the Neymann-Pearson most powerful test of size 0.03 rejects H_0 if $\frac{f_1(x)}{f_0(x)} \geq 3$, then the power of the test is

(A) 0.1 (B) 0.88 (C) 0.12 (D) 0.72

7) Let A and B be two square matrices. Then which of the following is not true?

(A) If λ is an eigen value of A, then λ is an eigen value of A^T.

(B) If λ is an eigen value of $A^{-1}B$, then λ is an eigen value of $B^{-1}A$.

(C) If A is an orthogonal matrix, then $|A| = \pm 1$.

(D) If λ is an eigen value of A, λ^{-1} is not an eigen value of A^{-1}.

8) The index of the quadratic form $Q(x, y, z) = x^2 - 2y^2 + 3z^2 - 4yz + 6zx$ is

(A) 1 (B) 3 (C) 2 (D) 0

9) Which of the following set of vectors are linearly independent?

(A) [3,1,-4], [2,2,-3], [0,-4,1] (B) [1,-7,9], [2,1,5], [8,-11,33]

(C) [1,0,0], [1,1,1], [0,0,1] (D) [1,2,3], [3,-2,1], [1,-6,-5]

10) Let $[x]$ is the greatest integer not exceeding x. Then $\int_0^4 x\mathrm{d}([\mathrm{x}] - \mathrm{x})$ is

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{3}{2}$

11) Which of the following is a point of minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$?

(A) (1,2) (B) (-1,2) (C) (1,-2) (D) (-1,-2)

12) $\lim_{n\to\infty} \sqrt[n]{3} = \ldots$

(A) $\frac{1}{2}$ (B) 1 (C) 0 (D) None of these

13) Which of the following is not true?

(A) $|A^T| = |A|$ (B) If A has a row of zeros, then $|A| = 0$

(C) If two rows of A are interchanged, then the determinant of the resulting matrix is $-|A|$.

(D) If A is an n-squared matrix, $|kA| = k^n|A|$

14) Let $\{X_n,\ n \geq 1\}$ be a sequence of uncorrelated r.vs with mean zero and variance 1. Then which of the following is true?

(A) $\{X_n, n \geq 1\}$ is a Markov chain (B) $\{X_n, n \geq 1\}$ is strictly stationary

(C) $\{X_n, n \geq 1\}$ is weakly stationary (D) $\{X_n, n \geq 1\}$ is evolutionary

15) Let $\{X_n, n \geq 0\}$ be a Markov chain with states 0,1,2 and one step t.p.m. $P = \begin{bmatrix} 0.75 & 0.25 & 0.0 \\ 0.25 & 0.5 & 0.25 \\ 0.0 & 0.75 & 0.25 \end{bmatrix}$. and the initial distribution $P(X_0 = i) = \frac{1}{3},\ i = 0, 1, 2$. Then $P(X_0 = 2, X_1 = 1, X_2 = 2)$ is

(A) $\frac{3}{64}$ (B) $\frac{1}{64}$ (C) $\frac{1}{16}$ (D) $\frac{3}{16}$

16) Which of the following is not true?

(A) In SRSWR, sample mean is an unbiased estimate of population mean.

(B) In SRSWR, variance of sample mean is $\frac{\sigma^2}{n}$.

(C) In SRSWOR, the probability of a specified unit being selecting at any draw is $\frac{n}{N}$.

(D) In SRSWOR, the sample mean is not an unbiased estimate of population mean.

17) Let X be an auxiliary variate and Y be the characteristic under study. Then the ratio estimate is always less precise if

(A) Coefficient of variation of X is equal to coefficient of variation of Y.

(B) Coefficient of variation of X is equal to twice coefficient of variation of Y

(C) Coefficient of variation of X is equal to half of the coefficient of variation of Y

(D) Coefficient of variation of X is equal to one third of the coefficient of variation of Y

18) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N_p(\mu, \Sigma)$ and let $\bar{X}$ be the sample mean and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})(X_i - \bar{X})'$. Then which of the following is not true?

(A) $\bar{X}$ and S^2 are jointly sufficient for μ and Σ.

(B) $\bar{X}$ and S^2 are independent.

(C) The maximum likelihood estimate of Σ is $\frac{1}{n}S^2$.

(D) $\bar{X}$ is distributed as $N_p(\mu, \frac{1}{n}\Sigma)$.

19) Which of the following is not a characteristic function?

(A) $\phi(t) = \begin{cases} 1 - |t|, & |t| \le 1 \\ 0, & |t| > 1. \end{cases}$ (B) $\phi(t) = \frac{2}{1+cost}$ (C) $\phi(t) = e^{-|t|^{\alpha}}, 0 < \alpha \le 2$

(D) $\phi(t) = \frac{1}{2-e^{-|t|}}$

20) The mean recurrence time for the state 2 of the Markov chain $\{X_n, n \ge 0\}$ having state space $\{1, 2, 3, 4\}$ and one step t.p.m. $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ is

(A) $\frac{5}{3}$ (B) $\frac{5}{2}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

21) Which of the following is a contrast?

(A) $3T_1+T_2-3T_3+T_4$ (B) $T_1+3T_2-3T_3+T_4$ (C) $-3T_1-T_2+T_3+3T_4$ (D) $T_1+T_2+T_3-T_4$

22) The maximum likelihood estimators are necessarily:

(A) unbiased (B) sufficient (C) most efficient (D) unique

23) Let $F_n^*(x)$ be the empirical distribution based on a sample from a population with cumulative distribution function $F(x)$. Then which of the following is not true?

(A) $F_n^*(x)$ is unbiased (B) $F_n^*(x)$ is consistent (C) $F_n^*(x)$ is consistent and unbiased
(D) none of these

24) For testing $H_0 : \sigma = \sigma_0$ in a normal population $N(0, \sigma^2)$, critical region based on sample $X_1, X_2, \ldots, X_n$ is $\frac{\sum X_i^2}{K}$. For which alternative hypothesis, does this provide uniformly most powerful test?

(A) $\sigma \neq \sigma_0$ (B) $\sigma^2 = \sigma_0^2$ (C) $\sigma < \sigma_0$ (D) $\sigma > \sigma_0$

25) If the two lines of regression are $X = -\frac{1}{18}Y + l$; $Y = -2X + m$ and the mean of the distribution is at (-1,2), the values of l and m are:

(A) $l = \frac{8}{9}, m = -5$ (B) $l = \frac{8}{9}, m = -3$ (C) $l = -\frac{10}{9}, m = -4$ (D) $l = -\frac{8}{9}, m = 0$

PART - B

Each question carries **ten** *marks.*

26) Consider the collection of intervals $\mathcal{C} = \left\{ \left(\frac{1}{n+1}, \frac{1}{n}\right), \; n = 1, 2, \ldots \right\}$. Obtain the σ-field generated by the sets in $\mathcal{C}$.

27) Let $\{X_n\}$ be a sequence of independent random variables, defined by
$P(X_n = 0) = 1 - \frac{1}{n^r}$ and $P(X_n = n) = \frac{1}{n^r}, r \geq 2, \; n = 1, 2, \ldots$.
Show that $X_n \xrightarrow{a.s} 0$ where as $X_n \overset{r}{\nrightarrow} 0$ as $n \to \infty$.

28) Let $X_1, \; X_2, \; X_3, \; X_4$ be independent $N(0, 1)$ r.v.s. Show that $Y = X_1X_2 + X_3X_4$ has the distribution with pdf $f(y) = \frac{1}{2}e^{-|y|}; \; -\infty < y < \infty$.

29) Let X and Y be independent geometric r.vs. Show that $\min(X, Y)$ and $X - Y$ are independent.

30) Let $X \sim N(\mu, \sigma^2)$. Based on a sample of size n, show that $T = \frac{2}{n(n+1)} \sum_{i=1}^{n} iX_i$ is consistent for μ. Is T unbiased? Establish your claim.

31) If a sufficient statistic T exists for the family $\{f_\theta : \theta \in \Theta\}, \; \Theta = \{\theta_0, \theta_1\}$, then show that the Neymann-Pearson most powerful test is a function of T.

32) Define Galton-Watson branching process. Establish its Markov property. If $\{X_n, \; n = 0, 1, 2, \ldots\}, \; X_0 = 1$ is a branching process, show that $\{X_{rk}, \; r = 0, 1, 2, \ldots\}$, where k is a fixed positive integer is also a branching process.

33) Describe double sampling and elaborate its application in ratio method of estimation.

34) What is confounding? Discuss the problem and analysis of complete confounding and partial confounding with the help of a 2^3 factorial experiment.

35) Define principal components. How do you determine them? Discuss its applications in statistics.

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